LMS Regional Meeting and Workshop

Loughborough University, 2–5 April 2024

Schedule

Regional Meeting (2nd April, Room U.0.20, Brockington Building)

	Tuesday
13:15 – 13:30	Opening of the LMS Meeting
13:30 – 14:30	Jonathan Bennett
14:30 – 14:45	Break
14:45 – 15:45	Oana Ivanovici
15:45 – 16:15	Tea
16:15 – 17:15	Christopher Sogge
17:15 – 18:30	Drinks Reception

Workshop (3rd – 5th April, Room SCH.1.01, Schofield Building)

	Wednesday	Thursday	Friday
09:30 – 10:30	Anke Pohl	Jonathan Hickman	Jan Rozendaal
10:30 – 11:00	Coffee	Coffee	Coffee
11:00 – 12:00	Veronique Fischer	Christopher Sogge	Daniel Grieser
12:00 – 13:00	Lunch	Lunch	Lunch
13:00 – 14:00	Discussion	Discussion	
14:00 – 15:00	Oana Ivanovici	Jeffrey Galkowski	
15:00 – 15:30	Tea	Tea	
15:30 – 16:30	Simon Myerson	Melissa Tacy	
16:30 – 18:30	Discussion	Discussion	

Abstracts for the Regional Meeting

Jonathan Bennett (University of Birmingham)

The Brascamp–Lieb inequalities and their adjoints

The Brascamp–Lieb inequalities are a generalisation of a range of important classical inequalities in mathematics. They include some very familiar inequalities, such as the Cauchy–Schwarz, Holder, and Young convolution inequalities. In addition to having a rich mathematical structure, the theory developed since their introduction in the 1970s has had far-reaching applications, from harmonic analysis to information theory, partial differential equations, number theory, combinatorial geometry, and even theoretical computer science. In this talk we give an introduction to the Brascamp–Lieb inequalities, and present a recent adjoint formulation that has found some unexpected applications to the theory of tomographic transforms (such as the X-ray and Radon transforms). These recent developments are joint work with Terence Tao.

Oana Ivanovici (Sorbonne Université)

Geometry and analysis for waves on domains

Christopher D. Sogge (Johns Hopkins University)

Curvature and harmonic analysis on compact manifolds

We shall explore the role that curvature plays in harmonic analysis on compact manifolds. We shall focus on estimates that measure the concentration of eigenfunctions. Using them we are able to affirm the classical Bohr correspondence principle and obtain a new classification of compact space forms in terms of the growth rates of various norms of (approximate) eigenfunctions.

This is joint work with Xiaoqi Huang following earlier work with Matthew Blair.

Abstracts for the Workshop

Veronique Fisher (University of Bath)

Towards quantum limits for subelliptic operators

The aim of the talk is to present recent developments of high frequency analysis for sub-elliptic operators. I will start with discussing why these questions are closely related to many aspects of harmonic analysis.

Jeffrey Galkowski (University College London)

The finite element method in high frequency scattering: non-uniform meshes defined by ray-dynamics

One of the most classical ways to numerically approximate the solution to high frequency scattering problems is the finite element method (FEM). In this method, one typically uses piecewise polynomials of some fixed degree p and a mesh-width h to approximate the solution. The fundamental questions is then: how should h be chosen (as a function of the frequency, k) so that the error in the numerical solution is small?

It has been known since the seminal work of Babuska and Ihlenberg that the natural conjecture of $hk \ll 1$ is not sufficient. Instead, one must require that $(hk)^{2p}\rho(k) \ll 1$ to maintain constant relative error, where $\rho(k)$ is the norm of the relevant resolvent. In this talk, we will show that this condition can be substantially weakened by using a non-uniform mesh which takes advantage of the fact that errors are concentrated in some regions rather than others.

Daniel Grieser (Universität Oldenburg)

Quasimodes for generalized semi-classical differential operators, Newton polygons, and blow-ups

We consider families P_h of differential operators on an interval that depend on a parameter $h \geq 0$ and degenerate as $h \to 0$. We consider the problem of constructing quasimodes, i.e. (families of) solutions u_h, $h > 0$, of $P_h u_h = O(h^\infty)$ as $h \to 0$. A classical example is the semi-classical Schroedinger Operator $P_h = h^2\partial^2 + V$ where $\partial = d/dx$ and V is a smooth function. If V is positive then quasimodes can be found using the standard WKB method. At zeroes of V additional difficulties arise (solved by Olver long ago) due to different scaling behavior near and away from the zeroes. Another classical example is Bessel's equation with parameter $\nu = 1/h$, where the behavior of solutions uniformly for large parameter and large argument is of interest (and well-known). One application of quasimode constructions is to find approximations of the spectrum of P_h for small positive h.

We construct, and give a precise description of, full sets of quasimodes for very general families $P_h = P(x, \partial_x, h)$ of any order, including the examples above as well as operators where the coefficients depend analytically on x and h, under a mild genericity hypothesis. The generality of the setup leads to a high degree of combinatorial and analytic complexity, which can be handled using an efficient representation of the data by Newton polygons and of the result in terms of iterated blow-ups and a suitable class of oscillatory-polyhomogeneous functions.

This is joint work with Dennis Sobotta.

Jonathan Hickman (University of Edinburgh)
Variable coefficient L^p local smoothing

I will discuss some recent work concerning variable coefficient extensions of L^p local smoothing estimates for the Schrodinger propagator. This can be thought of as a counterpart to a classical oscillatory integral operator bound of Bourgain (1991). Whilst Bourgain's result relies on studying Kakeya sets of curves, our L^p local smoothing result relies on studying Nikodym sets of curves. An important observation of Wisewell (2005) is that the Nikodym theory is surprisingly different from the Kakeya theory. Our work aims to further investigate and exploit these differences.

Part of an ongoing project with Shaoming Guo, Marina Iliopoulou and Jim Wright.

Oana Ivanovici (Sorbonne Université)
Dispersive estimates for the wave equation outside a strictly general convex obstacle in $\mathbf{R}^3$

We prove sharp dispersive bounds for the wave equation outside strictly convex obstacles in $\mathbf{R}^3$.

Simon Myerson (University of Warwick)
Bounds for spectral projectors on the three-dimensional torus

We study L^2 to L^p operator norms of spectral projectors for the Euclidean Laplacian on the three-dimensional torus, in the case where the spectral window is narrow. This builds on the classical result of Sogge and is related to the size of of L^p norms of eigenfunctions of the Laplacian, which were estimated by Bourgain. We prove new cases of our previous conjecture concerning the size of these norms. We use methods from number theory: the geometry of numbers, the circle method and exponential sum bounds due to Müller.

This is joint work with Pierre Germain.

Anke Pohl (Univerität Bremen)
Divisor of the Selberg zeta function with unitary representations

The classical Selberg zeta function is a mediator between spectral entities and dynamical entities of hyperbolic surfaces, as it is defined by means of the geodesic length spectrum and encodes in its zeros the spectral parameters of the Laplacian of the considered hyperbolic surface. We will consider the Selberg zeta function of infinite-area, geometrically finite hyperbolic orbisurfaces with twists by finite-dimensional unitary representation and hence for vector-valued situation. We will present a factorization formula in terms of the Weierstrass product of the Laplace resonances, Barnes G-functions, gamma functions and the singularity degrees of the representation. Similar to the classical, untwisted case, this provides a spectral and geometric interpretation of the zeros and poles of the Selberg zeta function, but this time by spectral and geometric entities of the orbisurface and by the representation. We will see that this factorization formula generalizes the factorization result by Borthwick, Judge and Perry to hyperbolic orbisurfaces with orbifold singularities as well as to unitary twists. We will further see that the presence of orbifold singularities yields a separate, previously unobserved contribution to the factorization formula, even in the untwisted case.

This is joint work with Moritz Doll.

Jan Rozendaal (Polish Academy of Sciences)

Hardy spaces for Fourier integral operators

It is well known that the wave operators $\cos(t\sqrt{-\Delta})$ and $\sin(t\sqrt{-\Delta})$ are not bounded on $L^p(\mathbf{R}^n)$, for $n \geq 2$ and $1 \leq p \leq \infty$, unless $p = 2$ or $t = 0$. In fact, for $1 < p < \infty$ these operators are bounded from $W^{(n-1)|\frac{1}{p}-\frac{1}{2}|,p}(\mathbf{R}^n)$ to $L^p(\mathbf{R}^n)$, and the exponent $(n-1)|\frac{1}{2}-\frac{1}{p}|$ cannot be improved. This phenomenon is symptomatic of the behavior of Fourier integral operators, a class of operators which includes the solution operators to smooth variable-coefficient wave equations.

In this talk, I will introduce a class of Hardy spaces $\mathcal{H}^p_{FIO}(\mathbf{R}^n)$, for $1 \leq p \leq \infty$, on which suitable Fourier integral operators are bounded. These spaces also satisfy Sobolev embeddings that allow one to recover the optimal boundedness results for Fourier integral operators on $L^p(\mathbf{R}^n)$.

In fact, the invariance of these spaces under Fourier integral operators allows for iterative constructions that are not possible when working directly on $L^p(\mathbf{R}^n)$, and such constructions can in turn be used to solve wave equations with rough coefficients. The spaces also provide a framework to capture improved local smoothing estimates for wave equations.

Christopher D. Sogge (Johns Hopkins University)

Improved Strichartz estimates for Schrödinger's equation on compact manifolds of nonpositive curvature

We are able to give logarithmic improvements of the universal Strichartz estimates for solutions of Schrodinger's equation on compact manifolds of Burq, Gerard and Tzvetkov. We use bilinear techniques developed by Tao–Vargas–Vega in their work on parabolic restriction problems along with an idea going back to Bourgain in his work on oscillatory integrals and Kakeya problems. This work is also closely related to our work on quasimode estimates. It is joint work with Xiaoqi Huang, which follows our earlier joint work with Matthew Blair.

Melissa Tacy (University of Auckland)

A quasimode approach to spectral multipliers

A central question of Euclidean harmonic analysis is; when does a multiplier

$$Mf = \mathcal{F}^{-1}\left[m(\cdot)\mathcal{F}[f](\cdot)\right]$$

defined as an operator $L^2 \to L^2$ extend to a bounded operator $L^p \to L^q$? The Bochner-Riesz multipliers where

$$m_R(\xi) = \left(1 - \frac{|\xi|^2}{R}\right)_+^{\delta}$$

are one well-known example of these type of operators. On manifolds we can consider analogous questions about whether spectral multipliers $M(\sqrt{\Delta})$ are bounded. Such questions have been long known to be connected to the growth properties of quasimodes (approximate solutions) to the eigenfunction equation $\sqrt{\Delta}u = \lambda u$. In this talk we will see how we can formalise the relationship between growth properties of quasimodes and boundedness of spectral multipliers and use the relationship to obtain new results about the latter.